

# Cognitive Zoom: From Object to Path and Back Again

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**Abstract.** This paper posits the usefulness of mental shifts of scale and perspective in thinking and communicating about spatial relations, and describes two experimental techniques for researching such cognitive activities. The first example involves mentally expanding a hand-sized piece of entangled string, a knot, so that following a portion of the string into a crossing resembles the act of walking along a path and over a bridge. The second example involves transforming experience and conceptions of the large-scale environment to small-scale representations through the act of mapmaking, and then translating the map to depictions of street-level views. When used in the context of clinical research methodologies, these techniques can help to elicit multimodal expressions of conceived topological relationships and geographical detail, with particular attention to individual differences.

## 1 Introduction

As advancements in computer graphics and communication technologies enable realization of geographic information systems (GIS), researchers are identifying needs to better understand how people perceive, represent, and communicate geographical information in their processes of constructing geographical knowledge. Researchers take varying approaches when applying results of cognitive studies to technology development and when using technologies as supports for cognitive studies. Additionally, some researchers seek to identify and formalize processes of human reasoning for embodiment within a computational system, while other researchers envision computational systems as information-supply and decision-support tools. In the latter views the human intelligence resides in the person using the machine rather than the machine itself, and the emphasis for system development is on the nature and range of functionality and its presentation in the human interface. The varying purposes lead to varying modes of inquiry, analysis, and interpretation.

Spatial cognition researchers employ different methods for different settings (such as the environment or the laboratory), and different methods for studying different aspects of spatial phenomena (Spencer et al. 1989). Generally, however, researchers aim to elicit and describe human perception and cognition of space. The descriptions may be verbal, pictorial, numerical, or logical. Nevertheless, patterns that emerge through the descriptions lead to theorizing and to further questions for research. Among spatial cognition researchers interested in GIS, high-priority research agendas include conceptualizations of geographical detail, understandings of dynamic phenomena and representations, and roles of multiple modalities for thinking about space and spatial relations (Mark et al. 1999).

The terms “geographic information” and “geographic knowledge” imply cognition of large-scale spaces. At this point our understanding of this sort of cognition is insufficient to say whether the same or similar processes may pertain to cognition of space at other scales:

It is important to recognize the distinction between geographical space and space at other scales or sizes. Palm-top and table-top spaces are small enough to be seen from a single point, and typically are populated with manipulative objects, many of which are made by humans. In contrast, geographical or large-scale spaces are generally too large to be perceived all at once, but can best be thought of as being transperceptual..., experienced only by integration of perceptual experiences over space and time through memory and reasoning, or through the use of small-scale models such as maps. Some of our discussions of geographical cognition might not apply to spatial cognition at other scales (ibid., p. 748).

However, members of an earlier survey characterized research in spatial cognition in terms of a developmental connection between “fundamental concepts of space,” implicitly at the scale of the human body, and conceptions of environments at geographic scales:

Researchers in spatial cognition focus on two kinds of conceptual growth: ...development of fundamental concepts of space, and the further differentiation and elaboration of these concepts into the development and representation of large-scale environments (Hart and Moore 1973, p. 248).

More work is needed to ascertain the presence and nature of such varying concepts and to understand whatever relations may exist between them.

There is a wide range of detail at the geographic scale. Just as it includes differences in degree as profound as those between cities and countries, the human scale ranges from palm-size and table-size to distances reachable through the course of a comfortable walk.<sup>1</sup> How common-sense notions of space may relate to faculties for spatial cognition at other scales remains a question. In fact, researchers interested in such phenomena must address a host of questions, such as whether certain spatial processes emerge at specific scales, and what sensory forms most appropriately represent conceptions at different scales (Mark et al. 1999, p. 761). A question of particular concern here is how changes of scale may influence granularity or clarity of data.

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<sup>1</sup> (Montello 1993) reviews and elaborates classification schemes for such variances.

This paper posits the usefulness of mental shifts of scale and perspective in thinking and communicating about spatial relations, and describes two examples of experimental approaches for researching such cognitive activities. The first example involves mentally expanding a hand-sized piece of entangled string, a knot, so that following a portion of the string into a crossing resembles the act of walking along a path and over a bridge. These degrees of detail are arguably within the same scale, that of the human body. The second example involves transforming conceptions of the large-scale environment to small-scale representations through the act of mapmaking, and then translating the map to depictions of street-level views. These degrees of detail concern different spatial scales, the geographic and that of the human body. The example also involves detailed translations within the pictorial scale.

An important distinction pertains to both of these examples, having to do with correspondences and variances between experiential and conceptual activity. Researchers usually discuss spatial scale with respect to the external world: depending on the classification scheme, the geographic scale may include the planet, countries, or cities; the environmental scale may include cities, town squares, or rooms in buildings; the human scale may include town squares, rooms, or table-tops; and so on. Conceptions and representations of the world may or may not maintain correspondences to these particulars. Here we explore shifts of scale that pertain in different ways to the external world and to internal worlds. Much of the conceptual activity involves changing perspectives and frames of reference.

The knot and mapmaking examples described in this paper stem from questions about how spatial thinking develops, and how understandings of basic spatial relations may be elaborated or otherwise changed through developmental processes. For now, the purpose of the two techniques is simply to elicit expressions of changing conceptions, for description and further study. The ant-crawling and mapmaking techniques are compatible with anthropological and clinical approaches to gathering data and a microgenetic approach to analyzing it (Berg and Smith 1985, Turkle 1984). The concern is with documenting richness and variety in people's ways of thinking and learning about space and spatial relationships (Turkle and Papert 1990). Thus the approach is to employ qualitative research methods, conduct longitudinal studies, and employ or develop experimental situations that will tend to evoke expressions of spatial conceptions.

These expressions take many forms: spoken description, including gestures and specific terminologies; written or typed descriptions and communications; hand-drawn sketches and diagrams; and constructed representations in various media (such as string and computer-based maps). Such representations become forms of data, supplementing the researcher's notes and audio and/or video recordings of discussions, work sessions, and open-ended interviews. All of these data are subject to protocol and microgenetic analysis (Ericsson and Simon 1984, Jacob 1987). The methods are appropriate to elucidation of higher-level conceptual structures rather than lower-level neurological structures.

## **2 Conceptual Elements**

Researchers across the range of scales in spatial cognition are concerned with the notion of conceptual elements. Their vocabularies differ somewhat; the elements may have to do with representations of objects in the environment and/or to relations between them.

Mental representations of geographical information seem to be constructed from elements, such as roads, landmarks, cities, land masses, the spatial relations among them, and the spatial relations of them to the larger units encompassing them (Mark et al. 1999, p. 757).

Kevin Lynch, an urban designer interested in “the possible connection between psychology and the urban environment” (Lynch 1984), articulated a version of this view by describing five elements of the “city image,” a form of mental imagery with which people remember aspects of the places they inhabit (Lynch 1960).<sup>2</sup> City images are highly individual but consistently include the general features of districts, edges, paths, nodes, and landmarks. These elements pertain to the structure of large-scale, built environments. Other aspects of urban experience, such as senses of sociability and aesthetics, are present in the data that Lynch and his colleagues collected but omitted from the focus on general structures. Lynch described the structures as existing both in the external environment and in the subjects’ conceptions of the environment. Inevitably, the internal world is more complicated than such a pristine characterization allows, however, and dilemmas arose as urban designers attempted to invert Lynch’s analytic process by using the sparse set of conceptual structures as a starting point in design. He later addressed this practical problem, reminding readers that the structural elements represent only part of urban experience and emphasizing the importance of including urban dwellers in design of the places they inhabit (Lynch 1984).

At the human scale, developmental psychologists have identified elementary spatial relationships that form the basis of growing conceptions (Piaget and Inhelder 1967).<sup>3</sup> These elements include relations of surrounding, proximity, separation, and order. Studies of knot-tying and understandings of knots have focused on articulating notions of surrounding and order (ibid., Strohecker 1991, Strohecker 1996a). These relationships take the forms of intertwining and of relative positions that demonstrate

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<sup>2</sup> The studies took place “at a time when most psychologists – at least, those in the field of perception – preferred controlled experiments in the laboratory to the wandering variables of the complicated, real environment. We hoped to tempt some of them out into the light of day” (Lynch 1984). The research established new ground leading to formation of the field of cognitive anthropology.

<sup>3</sup> It is important to clarify which aspects of Piaget’s voluminous work apply and which do not. His early notion of stage theory, which he all but abandoned in later writings, is not relevant to this discussion, nor are his notions of assimilation and accommodation. Rather, of interest are his techniques for activity-based experimental design and the clinical method with which Piaget and his colleagues were able to respond promptly and deeply to subjects’ productions and reports, forming new hypotheses within the course of an interview and ultimately collecting data beyond the relatively surface levels of attitudes and beliefs (Berg and Smith 1985, Piaget 1951, Turkle 1984). These methods continue to be useful to post-Piagetians who focus on individual differences and the importance of social context in learning (Turkle and Papert 1990, Harel and Papert 1991).

the property of between-ness. While simple knots concern relations of surrounding, more complex knots also involve combinations of forms, sequencing of production steps, and grouping types of productions. Thus thinking about complex knots becomes more broadly mathematical in this qualitative sense (Beth and Piaget 1966, Piaget and Inhelder 1967, Strohecker 1991).

Lynch's elements represent static environmental features, yet he and his colleagues focused on wayfinding in eliciting subjects' verbal, pictorial, and action-based descriptions of mental images. Had they not been so focused on notions of external structure, their data may have yielded formulations of more dynamic and/or relational properties (Lynch 1984, Strohecker 1999).<sup>4</sup> Just as Lynch focused on activity within the environment, and on people's translations of actions to conceptions, Piagetians considered conceptual elements as internalized actions (Gruber and Vonèche 1977). The developmental psychologists' elements are inherently process-based and pertain primarily to relations between spatial elements. In particular, relationships of surrounding are seen as mental analogs of the action of surrounding one thing by another.

### **3 Knots as Pathways**

Even simple knots can seem complicated, and learning to tie them can be challenging. The actions that generate a knot can be as fundamentally different as entwining separate pieces of string in modular fashion, convoluting two ends of a single piece of string, or leading one end of a string around, over, and under relatively fixed portions of the same strand. These methods typically require the use of two hands, but some talented tyers can single-handedly maneuver a string. Some can even toss a string into the air so that it descends into the form of a knot.

There are thousands of different knots, but each one demonstrates some general properties, understanding of which can help in learning to recognize and form new knots. In a study involving twenty-two children, aged ten to fourteen, subjects demonstrated a variety of ways of perceiving, describing, remembering, and effecting the relations of surrounding that characterize particular knots and "families" of knots (Strohecker 1996b). The children came from diverse cultural and socioeconomic backgrounds. They met in a workshop-style setting that grew in response to their interests and creations during the course of three months. Four work groups, consisting of about five children each, met once a week for hour-long sessions in which they worked with string as well as books and videos about knots.<sup>5</sup>

Three important elements of the research occurred through these free-form working sessions: the sessions formed a period of culture-building and of immersion in thinking about knots, so that the culminating interviews fit within a context that all the children shared; the working sessions, in their own right, generated data on thinking about knots; and in the course of the working sessions, many of the children

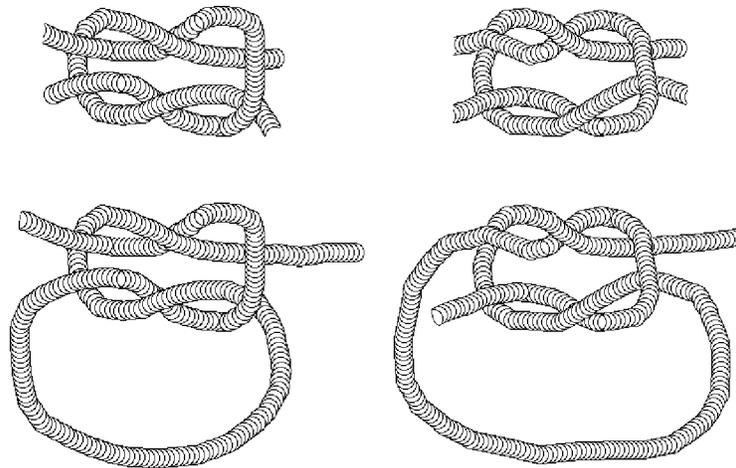
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<sup>4</sup> Indeed, the researchers were unable to completely extract this important aspect of urban experience: nodes are not merely static; as decision points, they pertain to the flow of human activity (Lynch 1960).

<sup>5</sup> For particulars, see (Strohecker 1991).

built up a relationship with the researcher that came to involve comfort and trust, which facilitated the “think aloud” nature of the relatively structured final interviews that generated the most comprehensive set of data. By the time of these interviews, the children had become familiar with ways of thinking and talking about knots. I asked the children to differentiate between two similar knots and to arrange a collection of about ten knots into groups according to whatever organizational scheme they thought appropriate. Here I focus on an aspect of these discussions which proved useful as children attempted to differentiate between two similar knots (Strohecker 1996a).

The knots known as the square knot and the thief knot seem identical when regarded locally. When seen in their entirety, however, it becomes clear that the knots are different:

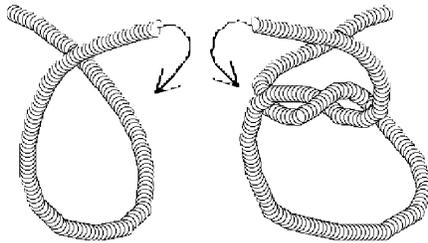


**Fig. 1.** The square knot and thief knot have different topologies but seem identical when viewed locally

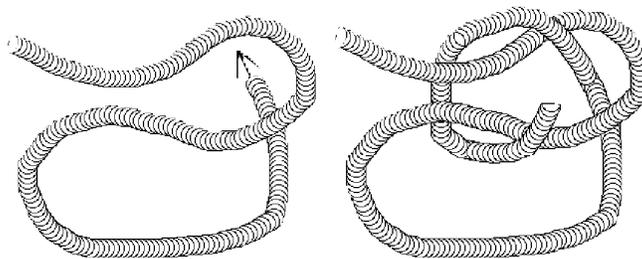
Children employed various ways of differentiating between these two knots. Some considered the configurations of the ends of the strings. However, they focused on the ends in different ways and used different terms even when focusing in the same ways: some children considered the positions of the ends relative to the entangled portion of the knot, while other children considered the end positions relative to the non-entangled loop. The entanglement was usually at the top of the odd-looking object we called a “knot,” but several children experimented with inverting and/or flipping the knots through the course of discussion. These moves augmented the vocabularies for describing differences between the configurations. Other children supplemented their vocabularies by comparing the two knots with other similar though different knots, such as the granny knot and the sheet bend.

Many of the children went beyond comparisons of the static objects, tying the square knot and the thief knot while describing the tying processes and then summarizing with a description of the finished knot. These descriptions tended to be richer in expressing details of the knots’ topologies: rather than relying on simple

terms and gestures to capture differences in positions of the ends (“in,” “out,” “up,” “down,” “across,” “straight,” “above,” “inside”), describing a tying process and its outcome requires specifying the “overs” and “unders” of the crossings that form the knot, and understanding their sequential relationships. Tying the square and thief knots also calls attention to a fundamental distinction from the outset: although both knots can be produced by entangling either one or two ends of a string, as the tyer may prefer, it is much easier to produce the thief knot by winding one end around itself, in snake-like fashion.

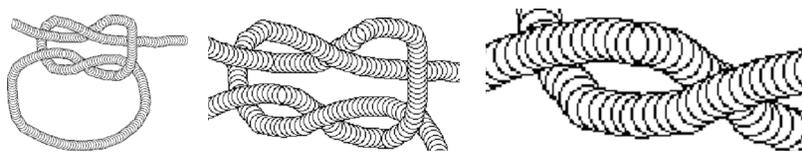


**Fig. 2.** A version of the usual method of producing a square knot



**Fig. 3.** A version of the usual method of producing a thief knot

Another effective technique in eliciting detailed expressions of conceptions of topological relationships in complex intertwinements is to prompt a shift of scale. Regarding a knot at its usual palm-top scale, children (and adults) often see a hopelessly entangled piece of string. However, they respond readily to the idea of imagining oneself as a tiny ant crawling along the string (Piaget and Inhelder 1967, Strohecker 1991). This mental shift of scale involves both space and time: the knot effectively enlarges as the observer assumes the new perspective, and subjects tend to dwell on each crossing for study and description.



**Fig. 4.** These illustrations are suggestive of the scale shift that subjects manage as they imagine themselves to be a “tiny ant” walking along a knot. However, it is important to remember that these shifts are mental, not external or pictorial. The thinking is also an active process, involving change over time as the “ant” moves along the string

Thus the shift helps people to focus on details of the knot's topology, to make sense of the sequence of over/under relationships that form the knot, and to notice other relevant relations of surrounding and proximity. The shift has the effect of transforming the experience of regarding the knot to that of a leisurely walk, invoking conceptions, language, and multimodal representations appropriate to that scale. For example, people engaging in this thought experiment sometimes refer to over-crossings as "bridges" and under-crossings as "underpasses."

Wayfinding is usually defined as finding a route between two points. The crawling-ant technique is a simpler exercise involving path-following by maintaining focus on a single, though moving, point. In this way it is more comparable to "route planning":

Route planning is modelled as a process of finding the overall direction and topology of the path, then filling in the details by deciding how to go around barriers (Mark et al. 1999, p. 758).<sup>6</sup>

Nevertheless, ant-crawling, route planning, and wayfinding are relevant to the same scale, that of a comfortable human walk, and the ant technique is effective in eliciting multimodal representations that describe the basic spatial relations. Subjects express these conceptions through speech, including word choice and varying intonations; hand gestures; coarser body language and movements; drawings; and written descriptions, including metaphors and narratives (Strohecker 1991).

#### **4 Pathways as Transformers of Scale and Perspective**

"WayMaker" is a graphical software tool with which researchers can prompt similar shifts of scale, along with changes of perspective and frame of reference (Strohecker and Barros 2000). Representations of Lynch's elements become facilities with which subjects construct map-like images of the structure of urban environments. In so doing they invoke conceptions at the geographic scale and translate them for diagrammatic representation. The resulting constructions are overviews of the large-scale domain, consisting of representations that may be abstract symbols (triangles for landmarks, lines for paths, etc.) or depictions of details of particular places (substituting towers for triangles, textured paths for lines, etc.).

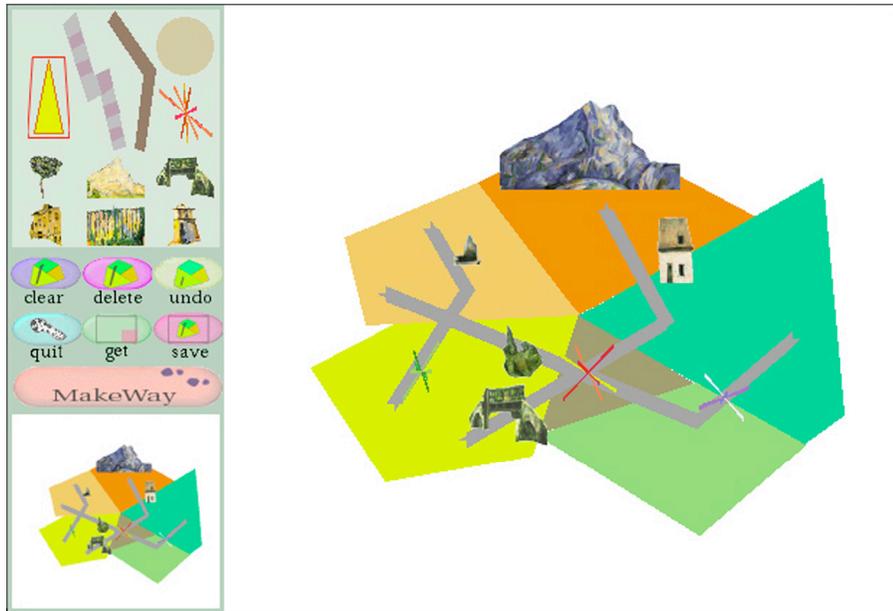
Not surprisingly, subjects vary in their strategies for placing these elements relative to one another. Some people form a "big picture" by creating a collection of districts and then placing smaller elements within them, while other people concentrate on a network of pathways; some people pay particular attention to landmarks, and so on. Thus the subject creates an individualized frame of reference that can act as a conceptual anchor when comparing the map to additional, shifted-scale representations of the environment. These shifts become apparent in WayMaker's second mode.

A miniature version of the map remains visible while the larger version disappears, as it is replaced by displays of street-level scenes along a chosen pathway. These

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<sup>6</sup> Indeed, "barrier" and "obstacle" are among the terms subjects used to describe under-crossings.

scenes appear one after the other, like a flip-card animation, illustrating rather than simulating the visual experience of a virtual world.



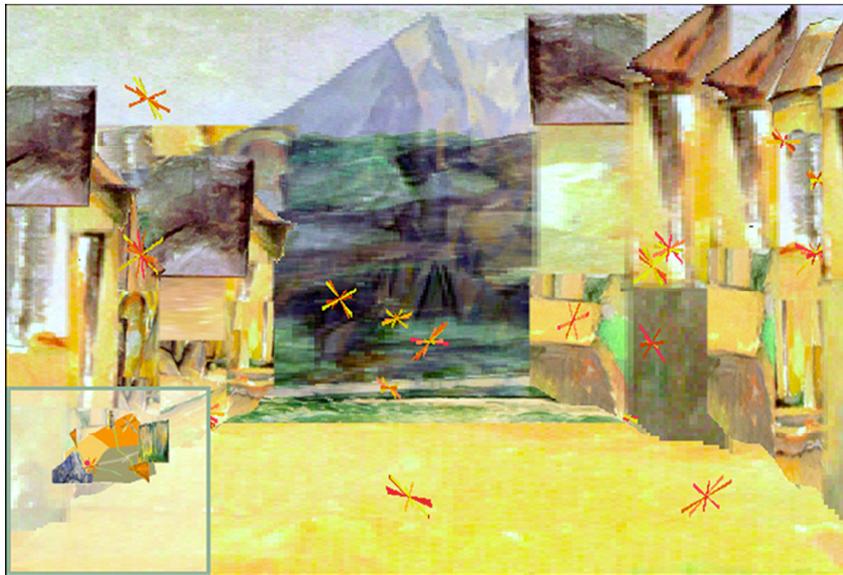
**Fig. 5.** The WayMaker construction screen presents representations of Lynch’s “elements of the city image” (upper left). Users position chosen representations with respect to one another in order to form a map of a large-scale environment (at right, echoed in the miniature at lower left)

The represented scale becomes that of a stroll through various districts. Details are consistent with the subject’s placement and characterization of the structural elements. They appear along the pathways and in the distance according to their relative placements on the map, moving forward and appearing around bends in the path to approximate perspective in the street-level views. Thus the second mode of the software maintains spatial relationships among the elements but effects shifts of representation, view, and scale.

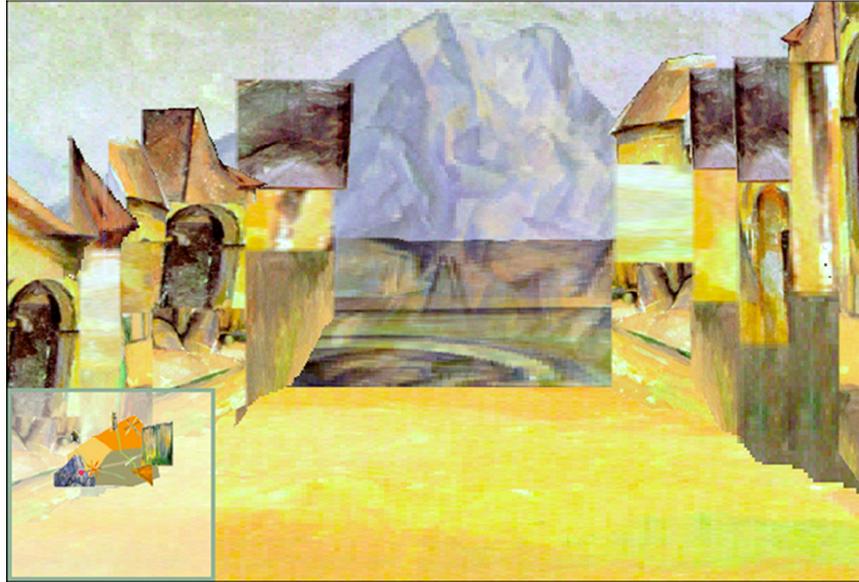
The manner of representation for the scenes is impressionistic and painterly in the current version of the software. The program composes scenes with images segmented from paintings by Paul Cézanne. We interpreted a similarity of purpose between our effort and Cézanne’s: many of his paintings represent studies of environmental structure and of relationships between built and natural environments (Machotka 1996). Additionally, we hoped that the appeal of this distinctive imagery would survive the scene-generation technique, which is fairly crude in this early prototype of the mapping tool. Though our approach to scene structure is different from Cézanne’s, WayMaker users often express enjoyment of the visual results, and this helps to sustain their continued experimentation with the prototype. In subsequent versions we hope to replace the Cézanne database with images drawn, photographed, or otherwise generated by the users themselves.



**Fig. 6.** WayMaker transforms the map to a series of street-level views along pathways through the environment. A moving dot on the miniature version of the map locates each view within the user's construction. The scenes are impressionistic but maintain spatial relationships among the districts, edges, paths, nodes, and landmarks



**Fig. 7.** The program composites scenes automatically, drawing from a database of images that consists of ground, back, and sky planes, as well as building bases, facades, and rooftops



**Fig. 8.** The program displays scenes along a pathway in the manner of a frame-by-frame animation

The subject can go easily back and forth between the construction and view modes. Thus there is an interesting combination of represented scales: the represented walk-scale of the views accompanies the diagrammatic scale of the map, which stems from the geographic scale (both external and represented) of the mapped environment. The geographic scale is both external and represented because the subject creates the map, an activity that involves drawing on direct experience of large-scale environments, conceptualizing features of such environments, and representing them diagrammatically. Although the subject composes the map as a computer-screen sized city layout, it is echoed in a smaller version that provides a basis for comparison with the street-level views. Thus there are arguably two diagrammatic scales for the map.

All of these represented levels are within the pictorial (palm-top/table-top) scale, yet the subject's act of creating the map differentiates use of WayMaker from mere map use.

Maps represent environmental and geographic spaces, but are themselves instances of pictorial space! ...the psychology of map use ... draw[s] directly on the psychology of pictorial space rather than on the psychology of environmental space (Montello 1993).

The psychology of map creation, however, draws on the psychology of environmental space. As map producers, WayMaker users must consider space at the environmental scale in order to compose a map for dynamic transformation. This exercise is comparable to that of other mapmakers who work with elemental representations, as for production of typical subway maps. We anticipate that WayMaker users' narrations and other expressions of expected and sensed experiences of the shifts of

scale, view, and representation will be deepened by the high degree of control afforded by the software: beyond constructing their own maps, subjects can interact in a variety of ways.

A red dot on the miniature map pinpoints the location of a given street view. The subject can click somewhere else on the miniature to define a route automatically for frame-by-frame animation. The program then calculates a route to this second point along the network of pathways, choosing turns when necessary based on which would constitute the shortest route. In the resulting dynamic display, the dot illustrates these programmatic decisions by moving along the paths, locating each new view within the environment. This manner of interaction is similar to path following. The subject may decide instead to click somewhere directly on a chosen path, so that the program does not need to make navigational decisions. This manner of interaction is more like the two-point technique of wayfinding. Alternatively, the subject can reposition the dot manually somewhere else along a pathway, to maximize control over the movement and effect a kind of fast-forward of the corresponding views.

Most importantly, the range, nature, and degree of interactivity encourage the subject to control shifts of his or her own mental states, invoking conceptualizations of environments, environmental features, and relations between environmental features formed through experience at environmental scales but actively represented and manipulated at smaller scales.

WayMaker extends Lynch's formulations by setting the environmental elements within a dynamic context, more consistent with the experience of a city and perhaps with the nature of its mental representations (Lynch 1984, Strohecker 1999). The program supports multiple perspectives and brings map construction and map reading together with cognition of direct environmental experience. Nevertheless, as a research tool, the program should be supplemented by other tools and techniques, just as Lynch's cohort combined multimodal representations and walk-and-talk personal interviews in their explorations of the city image.<sup>7</sup>

## 5 Further Work

These studies do not answer the question of whether the same conceptual structures are present in thinking about small-scale spaces and large-scale spaces. However, the ant-crawling technique highlights the facility with which people can shift from one conceptual level to the other. In so doing they may use structures at one level to

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<sup>7</sup> See (Strohecker 1999) for a discussion of the importance of participatory design contexts (Schuler and Namioka 1993, Henderson et al. 1998) as amelioration for WayMaker users' inversion of Lynch's analytic process. While it seemed that this inversion contributed to problems in urban design, Lynch contended that the real problem was the designers' omission of city dwellers from the design process. Understanding the dwellers' conceptions of their environments motivated his effort, and city dwellers' input in urban design was meant to return the richness of human experience to the relatively sparse results of the analytic process. Lynch contended that inhabitants of a city should be involved in its design. Similarly, extending WayMaker use beyond the prototype should involve participants in contexts shaped by the environmental structure design tool. Such modes might include neighborhood planning groups, multiuser virtual environments, and other domains that situate the tool use within appropriate social contexts.

illuminate or boost understandings at another. Further work is needed in order to document such strategies and examine relationships between spatial structures at different scales. Combined with qualitative research techniques, WayMaker may help in elucidating various forms of articulation of such conceptual elements.

Much of this sort of research is descriptive, and it is important not to generalize too quickly from responses to specific representations and depictions. A subject's difficulty of understanding may stem from a poorly interpreted image rather than inadequacy of some cognitive faculty, and poor interpretation of an image may result from bad design or an incomplete design process rather than some incapacity on the part of the viewer. Good design results from and depends on a cyclical, empirical process in which the designer generates ideas for various solutions, tests them with various people, and uses the results to inform subsequent solutions (Schön 1983). Interactive software applications pose particular, complex design problems pertaining to representation, sequencing of actions, and so on (DiBiase et al. 1992, Winograd 1996). Design of graphical displays and diagrammatic reasoning currently have low research priority (Mark et al. 1999), but may prove increasingly important as more geographic information systems emerge and as researchers seek to ensure that data guiding their design are sufficiently robust for this sort of generalization.

WayMaker needs further development in order to best support research in spatial cognition, though even in its current state it could help in generating data suggestive of research directions relevant to scale and other issues. The most important improvement may be development of new databases for the composited views, consisting of images created by the subjects themselves. Such personalization could help subjects get more deeply involved with the interactions, supporting the research potential to understand individual differences in the associated thinking (Papert 1980, Harel and Papert 1991, Kafai and Resnick 1996). Researchers focusing on varying levels and conditions of cognition recognize the significance of individual differences (Hutchins 1983, Kosslyn et al. 1984, Levinson 1996, Mark et al. 1999, Montello 1993, Turkle and Papert 1990). The population of subjects should vary across age, gender, and culture.

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